OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Write down the matrix for reflection in the y-axis. [1]

(ii) Write down the matrix for enlargement, scale factor 3, centred on the origin. [1]

(iii) Find the matrix for reflection in the y-axis, followed by enlargement, scale factor 3, centred on the origin. [2]

2 Indicate on a single Argand diagram

(i) the set of points for which $|z - (-3 + 2\text{j})| = 2$, [3]

(ii) the set of points for which $\arg(z - 2\text{j}) = \pi$, [3]

(iii) the two points for which $|z - (-3 + 2\text{j})| = 2$ and $\arg(z - 2\text{j}) = \pi$. [1]

3 Find the equation of the line of invariant points under the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}$. [3]

4 Find the values of A, B, C and D in the identity $3x^3 - x^2 + 2 \equiv A(x-1)^3 + (x^3 + Bx^2 + Cx + D)$. [5]

5 You are given that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -1 & 0 & 2 \\ 14 & -14 & 7 \\ -5 & 7 & -4 \end{pmatrix}$.

(i) Calculate $\mathbf{AB}$. [3]

(ii) Write down $\mathbf{A}^{-1}$. [2]

6 The roots of the cubic equation $2x^3 + x^2 - 3x + 1 = 0$ are α, β and γ. Find the cubic equation whose roots are 2α, 2β and 2γ, expressing your answer in a form with integer coefficients. [5]

7 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$ for all integers r. [2]

(ii) Hence use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(3r-1)(3r+2)}$. [5]

Section B (36 marks)

8 A curve has equation $y = \frac{2x^2}{(x-3)(x+2)}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. [3]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\frac{2x^2}{(x-3)(x+2)} < 0$. [3]

9 Two complex numbers, α and β, are given by $\alpha = 2 - 2\mathrm{j}$ and $\beta = -1 + \mathrm{j}$.

α and β are both roots of a quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$, where A, B, C and D are real numbers.

(i) Write down the other two roots. [2]

(ii) Represent these four roots on an Argand diagram. [2]

(iii) Find the values of A, B, C and D. [7]

10 **(i)** Using the standard formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, prove that

$$\sum_{r=1}^{n} r^2(r+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

(ii) Prove the same result by mathematical induction. [8]